The Third National Workshop on the SKA Project
The Italian Route to the SKAO Revolution
4 - 8 October

Advantages of the Inclusion of Globular Cluster Millisecond Pulsars in Pulsar Timing Arrays

(Based on the article “Including Millisecond Pulsars inside the Core of Globular Clusters in Pulsar Timing Arrays” currently under review)

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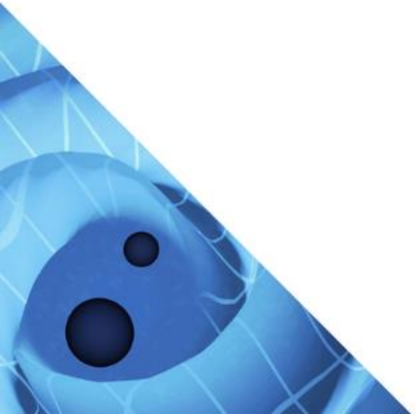


Pulsar Timing Arrays


ZW 096


NGC 7469

Pulsar Timing Arrays (PTAs) are the only detectors sensitive to ultra-low-frequency (in the range $\simeq 10^{-10} - 10^{-6}$ Hz) gravitational waves (GWs).

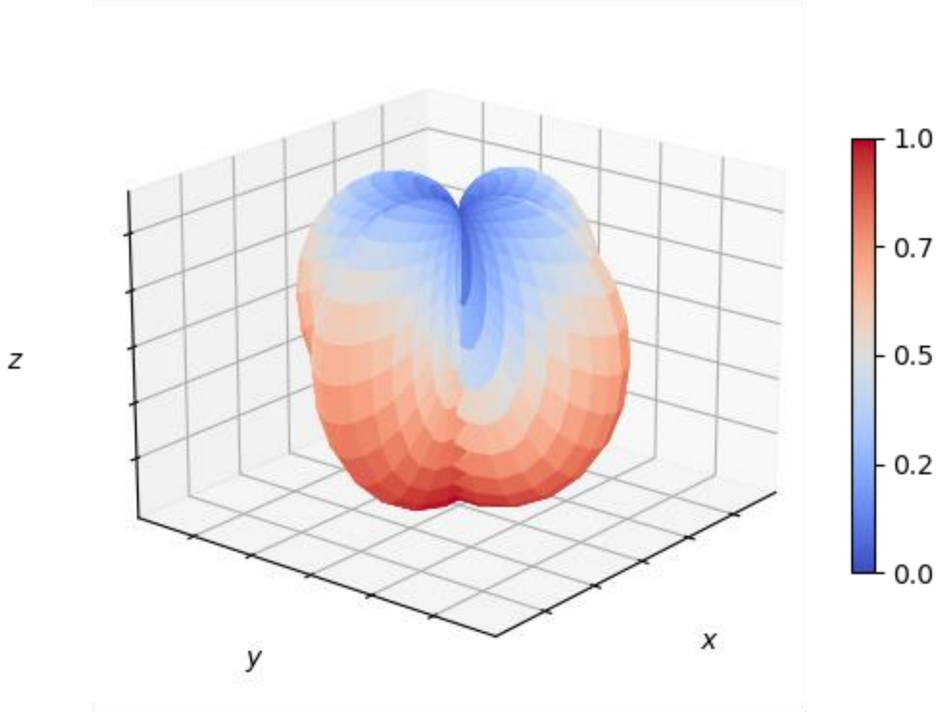
These GWs can be generated by sources relevant for astrophysics and cosmology, such as supermassive black holes (SMBHs) or cosmic strings, and form a gravitational wave background (GWB).

Almost all bright galaxies are believed to have an SMBH in their center. We have examples of merging galaxies in literature. Therefore, GWs emitted by SMBHBs are the main targets of the PTAs.

Credits (ZW 096): NASA/JPL-Caltech/STScI/H. Inami (SSC/Caltech)
Credits (NGC 7469): NASA, ESA, the Hubble Heritage (STScI/AURA)-ESA/Hubble Collaboration, and A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University)

Moore, C. J. ; Cole, R. H. ; Berry, C. P. L. Gravitational-wave sensitivity curves. Classical and Quantum Gravity, Volume 32, Issue 1, article id. 015014 (2015)

Continuous Gravitational Waves



GW induced timing residuals due to a single SMBHB:

$$r(f,\Omega^i) = \int_0^t dt \sum_{A=+,\times} F^A(\Omega^i)[h^A(t, x^i = 0) - h^A(t-\tau, x^i = \tau p^i)]$$

Antenna pattern function of the A polarization:

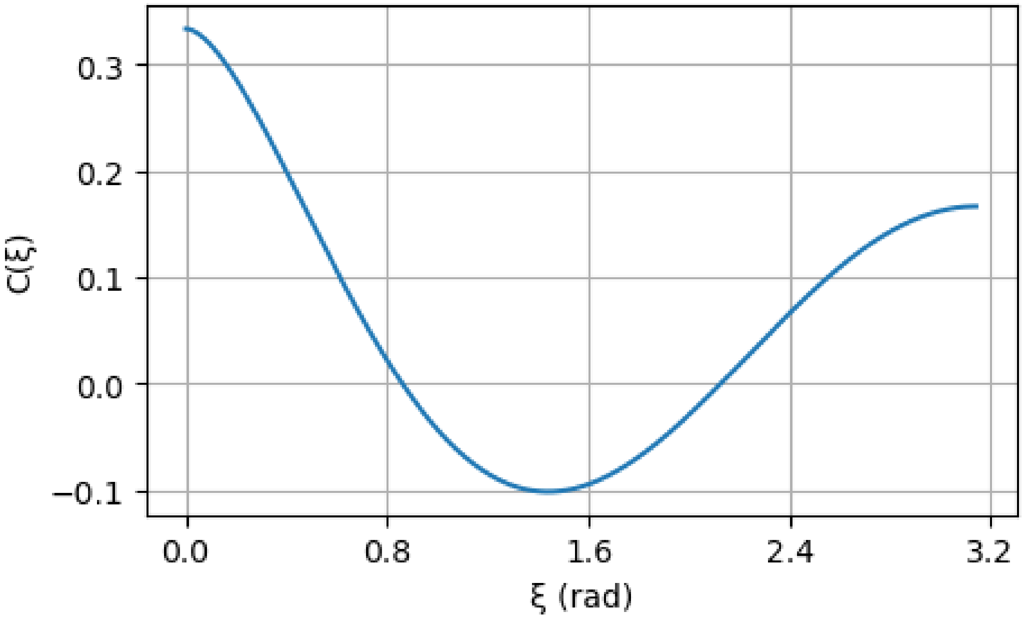
$$F^A(\Omega^i) = \frac{1}{2}\frac{p^i p^j e_{ij}}{(1+\Omega_i p^i)}$$

A polarization strain:

$$h^A(f,\Omega^i) = he^{i(k_\mu x^\mu + \alpha^A)}$$

Yunes, N.; Siemens, X. Gravitational Wave Tests of General Relativity with Ground-Based Detectors and Pulsar Timing Arrays. Living Reviews in Relativity, Volume 16, Issue 1, article id. 9, 124 pp. (2013)

Stochastic Gravitational Wave Background



GW induced timing residuals due to a population of SMBHB:

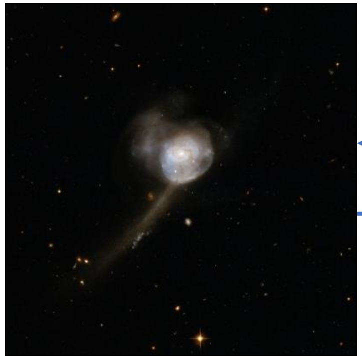
$$R\left(f,\Omega^i\right) = \int_{4\pi} d^2\Omega^i \int_{-\infty}^{\infty} df\, r\left(f,\Omega^i\right)$$

Hellings & Downs function describing the quadrupole cross-correlation between the GW induced timing residuals of two pulsars:

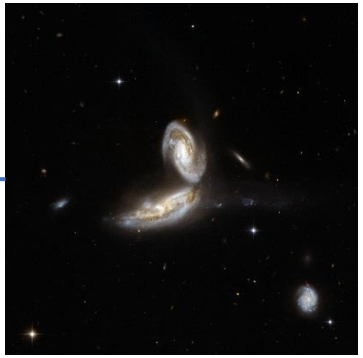
$$C(\xi) = \frac{1}{3}\left\{1 + \frac{3}{2}(1 - cos\xi)\left[ln\left(\frac{1 - cos\xi}{2}\right) - \frac{1}{6}\right]\right\}$$

Detweiler, S. Pulsar timing measurements and the search for gravitational waves. Astrophysical Journal, Part 1, vol. 234, p. 1100-1104 (1979)
Hellings, R. W. ; Downs, G. S. Upper Limits on the Isotropic Gravitational Radiation Background from Pulsar Timing Analysis General Relativity and Gravitation, Vol. 1, Classical Relativity (1983)

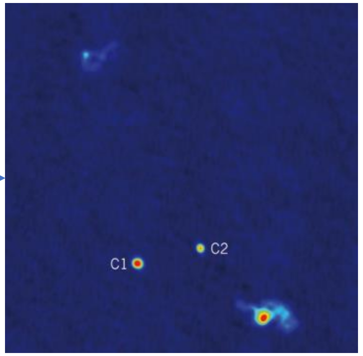
Current State of Gravitational Wave Detection



NGC 17



NGC 5331



Galaxy 0402+379

Some clues of the presence of a common red noise compatible with a GWB have already been found, but there is still not strong enough evidence for or against it.

PTA collaborations collected more than 12 years of pulsar timing data. Possible reasons behind no detection:

- more years of pulsar timing data are needed;
- more MSPs are needed;
- last-parsec problem is real and prevents the SMBHB coalescence.

Credits (NGC 5331): NASA, ESA, the Hubble Heritage Team (STScI/AURA)-ESA/Hubble Collaboration and A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University)
Credits (NGC 17): NASA, ESA, the Hubble Heritage Team (STScI/AURA)-ESA/Hubble Collaboration and A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University)

Taylor, G. A tight duo in a trio of black holes. Nature, Volume 511, Issue 7507, pp. 35-37 (2014)
Burke-Spolaor, S. et al. The Astrophysics of Nanohertz Gravitational Waves. The Astronomy and Astrophysics Review, Volume 27, Issue 1, article id. 5, 78 pp. (2019)
Arzoumanian, Z. et al., The NANOgrav 12.5 yr data set: Search for an isotropic stochastic gravitational-wave background. The Astrophysical Journal Letters, Volume 905, Issue 2, id.L34, 18 pp. (2020)

Step Back

The Hellings & Downs function comes from:

$$\langle z_a(t) z_b(t) \rangle = \int_{-\infty}^{\infty} df\, S(f) C(\xi)$$

$$C(\xi) = \int_{4\pi} \frac{d^2\Omega^i}{4\pi} \mathcal{K}\left(f, \Omega^i\right) \sum_{A=+,\times} F_a^A\left(\Omega^i\right) F_b^A\left(\Omega^i\right)$$

$$\mathcal{K}\left(f, \Omega^i\right) = \left[1 - e^{-2\pi i f \tau_a (1+\Omega_i p_a^i)}\right]\left[1 - e^{2\pi i f \tau_b (1+\Omega_i p_b^i)}\right]$$

$$\mathcal{K} \rightarrow 1$$

Interesting Case

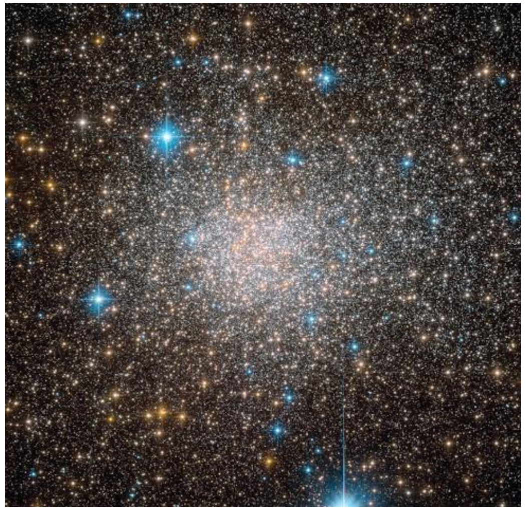
The pulsar terms of two MSPs are generally not correlated unless they are very close to each other so that they are perturbed almost simultaneously by GWs:

$$\Delta x^i \to 0 \Rightarrow \mathcal{K} \to 2$$

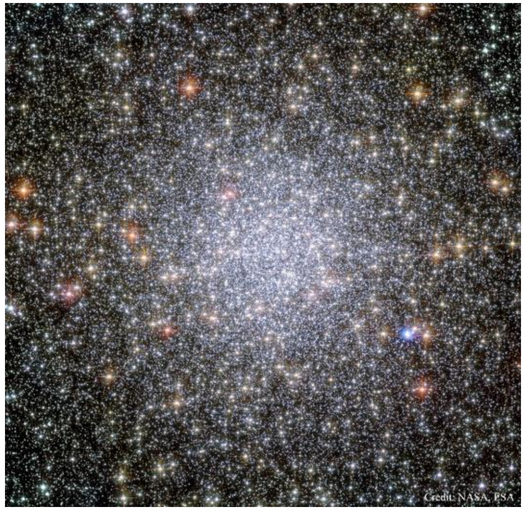
Therefore, in this case, the cross-correlation should be improved.

Michele Maggiore. Gravitational Waves: Volume 2: Astrophysics and Cosmology. Oxford Scholarship Online (2018)

Globular Clusters



Ter 5



47 Tuc

Globular clusters (GCs) can be a good testing ground for this effect since they host many MSPs within a small radius from their center. A couple of noticeable examples:

- Terzan 5 hosts about 15 MSPs within 0,16 arcmin from its center;
- 47 Tucanae hosts about 10 MSPs within 1,2 arcmin from its center.

Credits (Ter 5): F. Ferraro / NASA / ESA / ESO
Credits (47 Tuc): NASA, ESA, Hubble Heritage Team (STScI / AURA) Acknowledgment: J. Mack (STScI) and G. Piotto (U. Padova)

Pulsars in globular clusters. https://www.naic.edu/~pfreire/GCpsr.html
The 27 millisecond radio pulsars in 47 Tucanae. https://www3.mpifr-bonn.mpg.de/staff/pfreire/47Tuc/
Cadelano, M. et al. Discovery of Three New Millisecond Pulsars in Terzan 5. The Astrophysical Journal, Volume 855, Issue 2, article id. 125, 7 pp. (2018)

Simulation Details

GC:

- distance (Terzan 5);
- core radius (Terzan 5);
- number of MSPs (Terzan 5).

SMBHB population:

- position;
- chirp mass;
- number of SMBHB;
- emitted GW frequency.

MSP positions:

- $p^i = \left(\sin\theta_p \cos\phi_p \,, \sin\theta_p \sin\phi_p \,, \cos\theta_p\right)$

GW directions:

- $\Omega^i = (\sin\theta \cos\phi \,, \sin\theta \sin\phi \,, \cos\theta)$
- $m^i = (\sin\phi \,, -\cos\phi \,, 0)$
- $n^i = (\cos\theta \cos\phi \,, \cos\theta \sin\phi \,, -\sin\theta)$

Polarization tensors:

- $e^{+}_{ij} = m_i m_j - n_i n_j$
- $e^{\times}_{ij} = m_i n_j + n_i m_j$

Anholm, M.; Ballmer, S.; Creighton, Jolien D. E. ; Price, L. R. ; Siemens, X. Optimal strategies for gravitational wave stochastic background searches in pulsar timing data. Physical Review D, vol. 79, Issue 8, id. 084030 (2009)

Slowly-Evolving Binary Approximation

Generally, the GW frequency at the position of MSP and that at the position of the observer is not the same, because the SMBHB evolves during the pulse travel time from the MSP to the observer.

However, in most cases, the pulse travel time is much shorter than the evolution time of the SMBHB. Therefore, the frequency difference can be neglected.

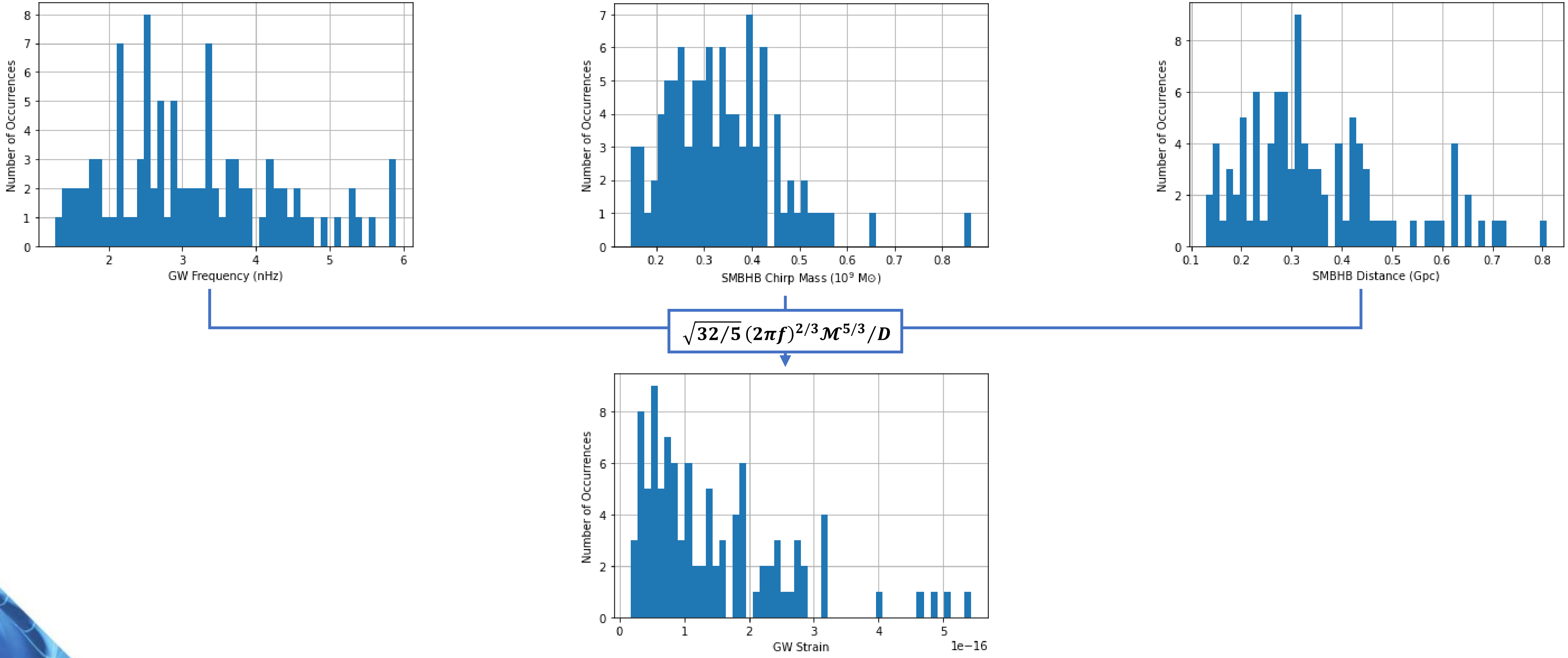
Pulse travel time:

- $\tau_p = 5{,}9\ \mathrm{kpc} = 1{,}9 \times 10^4\ \mathrm{yrs}$

Evolution time:

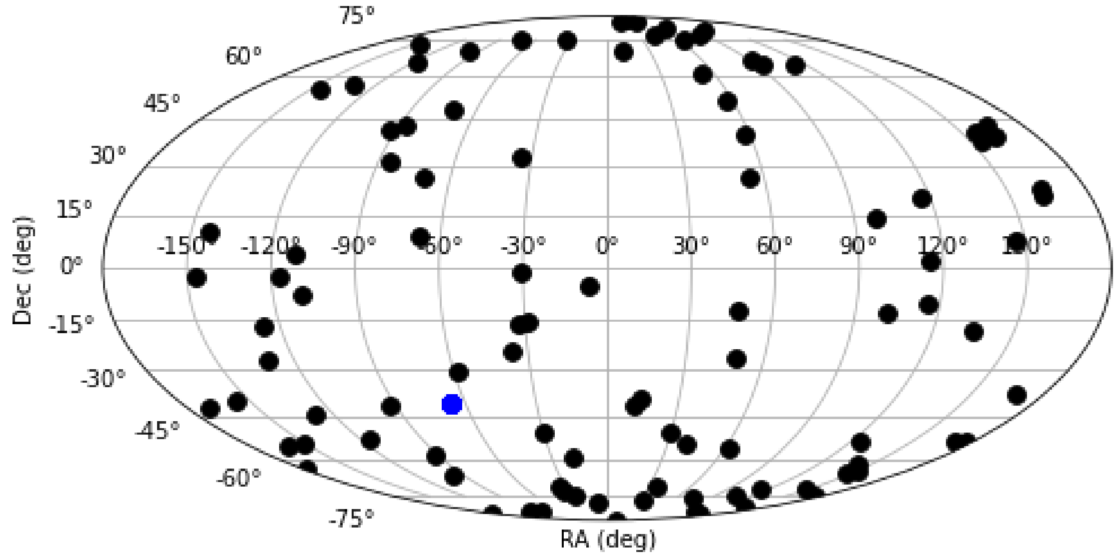
- $\tau_e = \frac{5}{256} c^5 (\mathcal{M}G)^{-5/3} \left[\omega^{-8/3}\right]_{\Delta\omega}$
- $\tau_e(10^9\ \mathrm{M}_\odot,\ 10^{-9}\mathrm{Hz} \rightarrow 10^{-8}\ \mathrm{Hz}) = 3{,}2 \times 10^6\ \mathrm{yrs}$

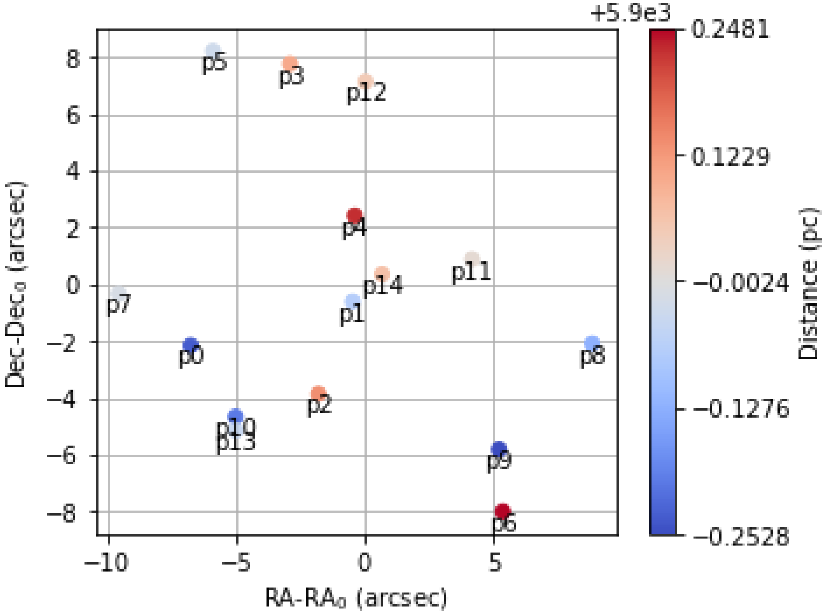
Parameter Distributions



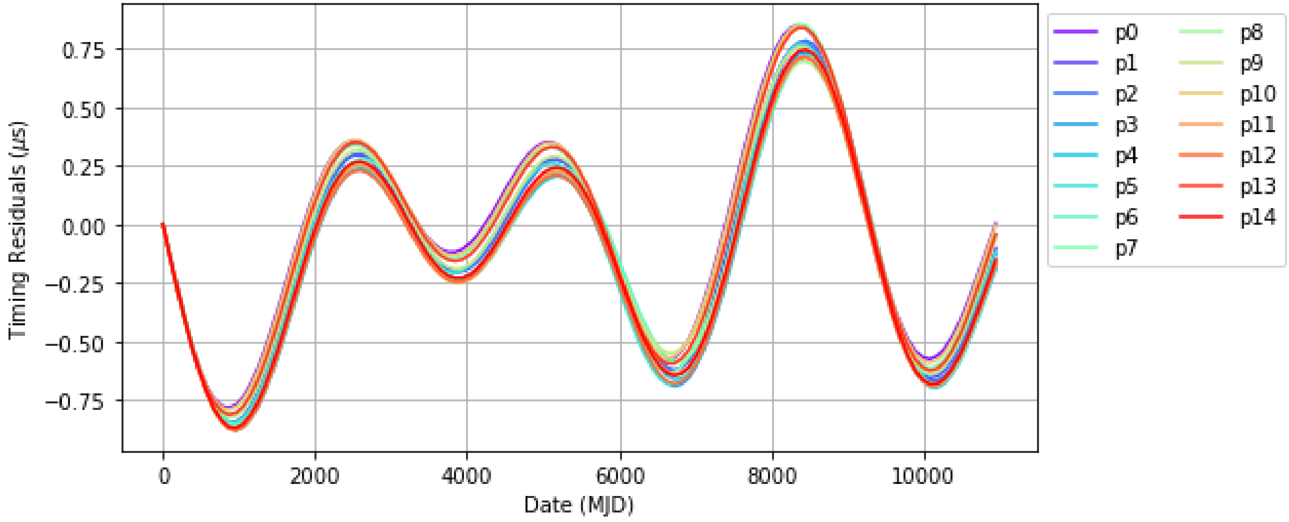
$$\sqrt{32/5}\,(2\pi f)^{2/3}\mathcal{M}^{5/3}/D$$

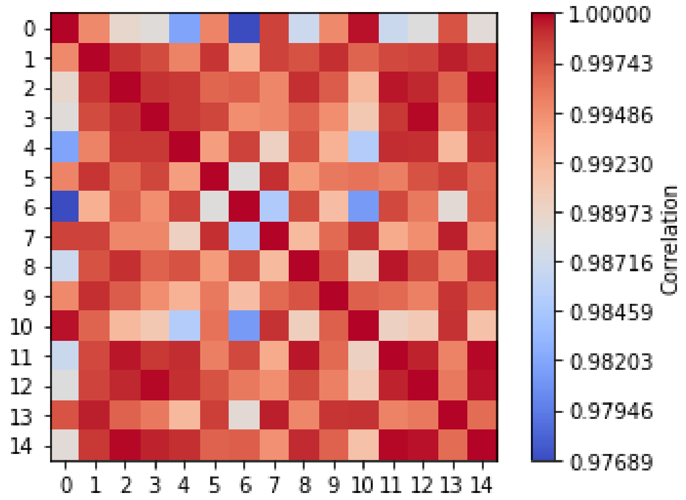
Position Distributions



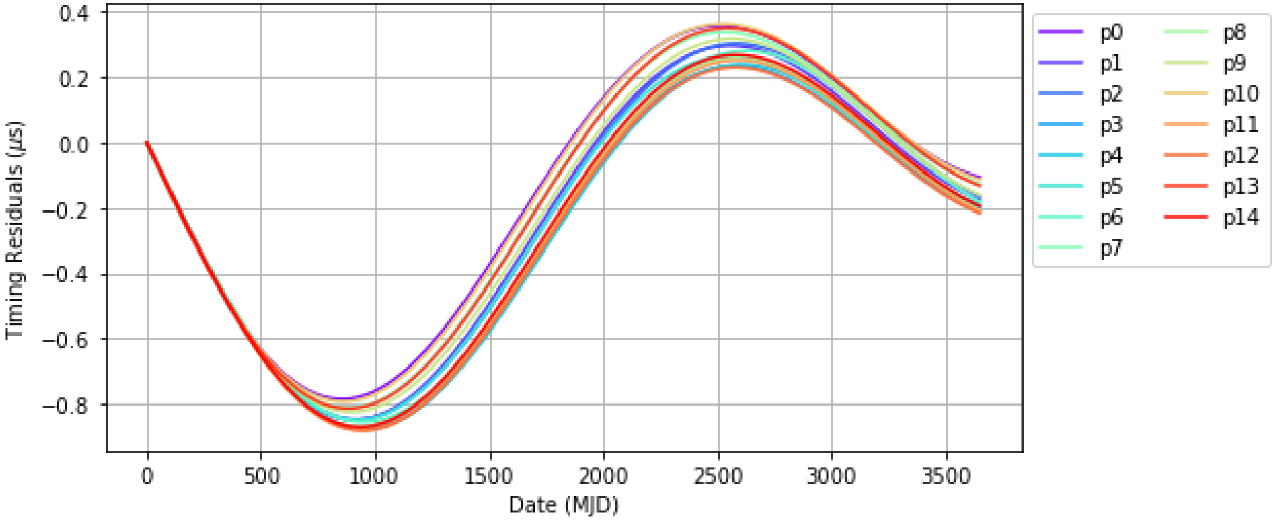


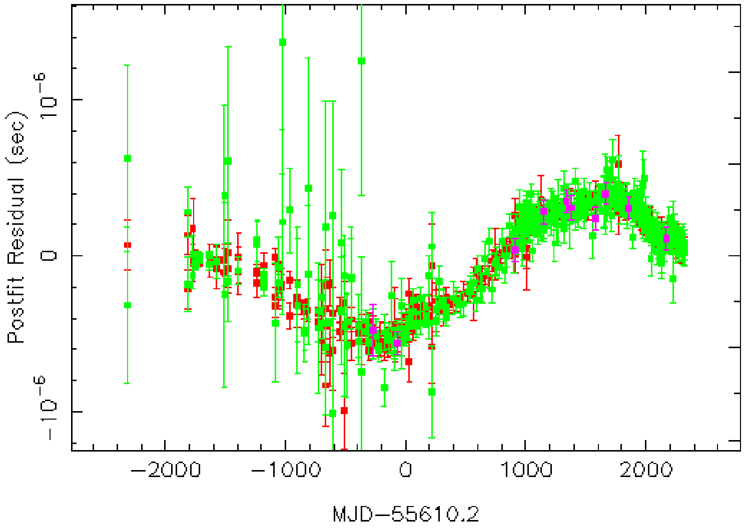
30-Year Timing Residuals





10-Year Timing Residuals



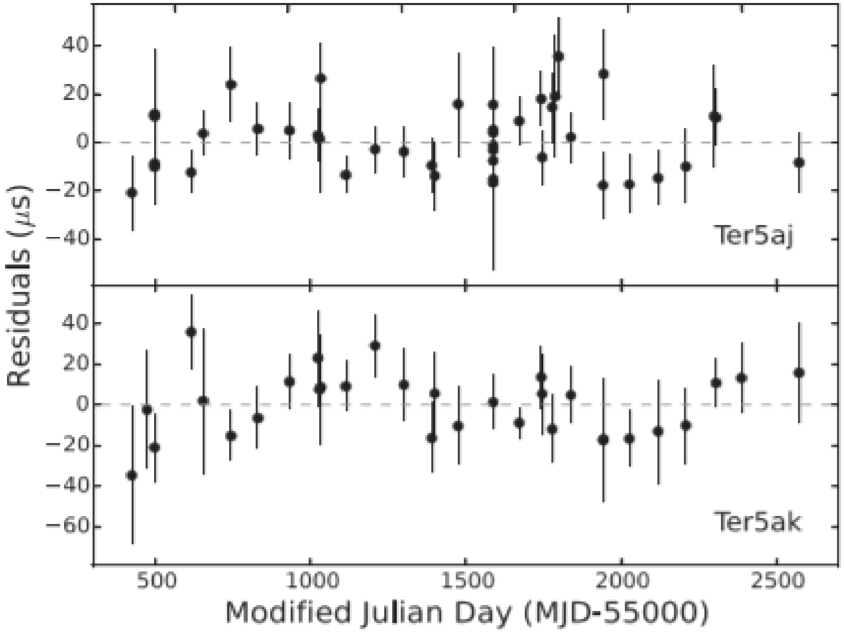


J1909-3744

NANOGrav 12.5-year data set. https://data.nanograv.org/

Hobbs, G. B. ; Edwards, R. T. ; Manchester, R. N. TEMPO2, a new pulsar-timing package – I. An overview. Monthly Notices of the Royal Astronomical Society, Volume 369, Issue 2, pp. 655-672 (2006)

Timing Accuracy



Timing the GC MSPs is very difficult. Their dynamics is influenced by the GC gravitational field as well as the motion of nearby stars.

In most cases the GC MSPs are further away than the PTA MSP, therefore, the dispersion measure is more significant.

It is difficult to obtain timing residuals below hundred nanoseconds.

Cadelano, M. et al. Discovery of Three New Millisecond Pulsars in Terzan 5. The Astrophysical Journal, Volume 855, Issue 2, article id. 125, 7 pp. (2018)

Conclusion

GW-induced timing residuals of GC MSPs, especially those in their cores, should be strongly correlated, and that correlation should, in principle, be detectable.

Currently, almost none of the GC MSPs (except B1821-24A, in M28) are included in PTAs. In the next ten years, this situation may change thanks to the next generation radio-telescopes, like the Square-Kilometre Array (SKA), and this would open the possibility of adopting the strategy proposed here.

Xiaojin L. Pulsar Timing with the Next Generation of Radio Telescopes. University of Manchester. https://www.research.manchester.ac.uk/portal/files/163047969/FULL_TEXT.PDF
Desvignes G. et al. High-precision timing of 42 millisecond pulsars with the European Pulsar Timing Array. Monthly Notices of the Royal Astronomical Society Volume: 458, Issue: 3, pp 3341-3380 (2016)
Kerr, M. et al. The Parkes Pulsar Timing Array Project: Second data release. Publications of the Astronomical Society of Australia, Volume 37, article id. E020. (2020)
Alam, M. F. et al. The NANOGrav 12.5 yr Data Set: Observations and Narrowband Timing of 47 Millisecond Pulsars. The Astrophysical Journal Supplement Series, Volume 252, Issue 1, id.4, 48 pp. (2021)

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Thanks for your Attention

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