The possible region of origin of the Cavezzo meteorite

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Southworth's and Hawkins' criterion

To classify meteors in streams, or in general to judge the degree of similarity of two orbits, the standard tool is Southworth's and Hawkins' orbital similarity criterion:

$$\begin{aligned}
D_{SH}^2 &= [e_2 - e_1]^2 + [q_2 - q_1]^2 + \left[2\sin\frac{I_{21}}{2}\right]^2 \\
&\quad + \left[\left(\frac{e_2 + e_1}{2}\right)\left(2\sin\frac{\pi_{21}}{2}\right)\right]^2 \\
\left[2\sin\frac{I_{21}}{2}\right]^2 &= \left[2\sin\frac{i_2 - i_1}{2}\right]^2 + \sin i_1 \sin i_2 \left[2\sin\frac{\Omega_2 - \Omega_1}{2}\right]^2 \\
\pi_{21} &= \omega_2 - \omega_1 \\
&\quad + 2\arcsin\left[\cos\frac{i_2 + i_1}{2}\sin\frac{\Omega_2 - \Omega_1}{2}\sec\frac{I_{21}}{2}\right].
\end{aligned}$$

The Earth-crossing condition

When meteoroids hit the Earth at ☊ we have

$$\omega + f = 0^\circ \rightarrow f = -\omega,$$

when this happens at ☋

$$\omega + f = 180^\circ \rightarrow f = 180^\circ - \omega;$$

in both cases, we have $r = 1$ au, implying that, at ☊ and ☋, respectively,

$$\begin{aligned} 1 &= \frac{a(1-e^2)}{1 + e\cos(-\omega)} \\ 1 &= \frac{a(1-e^2)}{1 + e\cos(180^\circ - \omega)}. \end{aligned}$$

How to characterize meteoroid orbits?

Therefore a meteoroid orbit is characterized by only 4 independently measurable quantities.

The problem has really only 4 dimensions, and the criterion by Southworth and Hawkins is 5-dimensional; does there exist a suitable set of 4 variables that would allow meteor stream identification in a "natural" way?

And, if so, would not it be better if these variables were directly deducible from observed quantities, without having to derive the orbital elements?

Note that the conventional orbital elements, that are necessary to compute D_{SH}, are the conserved quantities of the simplest problem of celestial mechanics, i.e. the 2-body problem.

New variables

Let is now consider a meteor. Its date and time give us Ω of the meteoroid's orbit since, if the the fall takes place at ☊, we have

$$\Omega = \lambda_{\oplus},$$

and if it takes place at ☋ we have

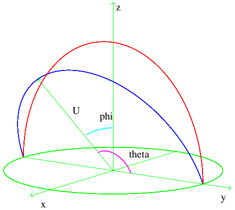
$$\Omega = \lambda_{\oplus} + 180^{\circ}.$$

We need three more quantities to completely characterize the orbit and we do not want to use a, e, i, ω because they are 4 quantities related by a constraint.

New variables

The geometric setup of Öpik's method suggests us the natural choice: U, $\cos\theta$ and ϕ.

What do these variables represent?



U is the modulus of the geocentric unperturbed velocity.

The angles θ and ϕ define the direction opposite to the geocentric radiant.

So, U, θ and ϕ are obtained from the directly measurable quantities that characterize an observed meteor.

Note that U and $\cos\theta$ are secularly invariant quantities for the Lidov-Kozai perturbation.

The new criterion

Valsecchi, Jopek & Froeschlé (1999) thus defined a new criterion for the similarity of meteoroid orbits:

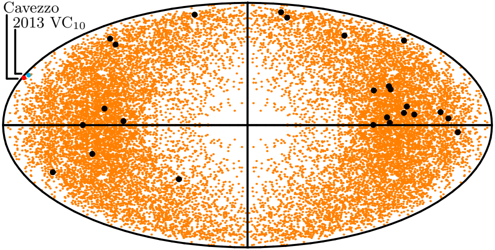
$$
\begin{aligned}
D_N^2 &= [U_2 - U_1]^2 + w_1[\cos\theta_2 - \cos\theta_1]^2 + \Delta\xi^2 \\
\Delta\xi^2 &= \min\left[w_2\Delta\phi_I^2 + w_3\Delta\lambda_I^2, w_2\Delta\phi_{II}^2 + w_3\Delta\lambda_{II}^2\right] \\
\Delta\phi_I &= \left[2\sin\frac{\phi_2 - \phi_1}{2}\right] \\
\Delta\phi_{II} &= \left[2\sin\frac{180^\circ + \phi_2 - \phi_1}{2}\right] \\
\Delta\lambda_I &= \left[2\sin\frac{\lambda_2 - \lambda_1}{2}\right] \\
\Delta\lambda_{II} &= \left[2\sin\frac{180^\circ + \lambda_2 - \lambda_1}{2}\right].
\end{aligned}
$$

The geocentric quantities

Applying the new criterion to Cavezzo and all the NEAs in NEODyS, the only reasonably close (but not too close) match is between Cavezzo and 2013 VC_{10}, a small NEA ($H = 24.8$) observed only at its discovery apparition.

	Cavezzo	2013 VC_{10}
U	0.216 ± 0.001	0.1818
θ (°)	22.96 ± 0.30	24.8358
ϕ (°)	175.90 ± 0.69	171.49
$\lambda_{\oplus}$ (°)	100.52311 ± 0.00001	104.986

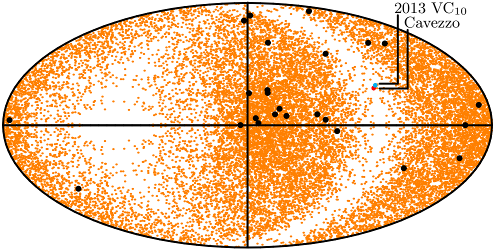
The radiant



Equal area projection of the sky centred on the apex of the Earth motion; coordinates: ecliptic longitude minus longitude of Sun, and ecliptic latitude.

Red dot: Cavezzo radiant; cyan dot: 2013 VC_{10} radiant; orange dots: radiants of simulated impactors of Chesley & Spahr (2004); black dots: radiants of the 20 meteorites listed in Granvik & Brown (2018).

The radiant



Same as before, but centred on the opposition. The isolation of Cavezzo and of 2013 VC_{10} is even more evident.

The heliocentric orbital elements

If 2013 VC_{10} is really the parent body of Cavezzo, the close match of ϖ and the differences in Ω, ω tell us that the separation has taken place some time ago.

	Cavezzo	2013 VC_{10}
a (au)	1.82 ± 0.22	1.56622
e	0.460 ± 0.063	0.365295
i (°)	4.0 ± 1.6	2.044
Ω (°)	280.52311 ± 0.00001	224.068
ω (°)	179.2 ± 4.8	240.264
ϖ (°)	99.7 ± 4.8	104.332
T (JD)	2458849.6 ± 0.5	2458808.1
q (au)	0.983 ± 0.001	0.9941
Q (au)	2.66 ± 0.41	2.1383